Graphical SLAM

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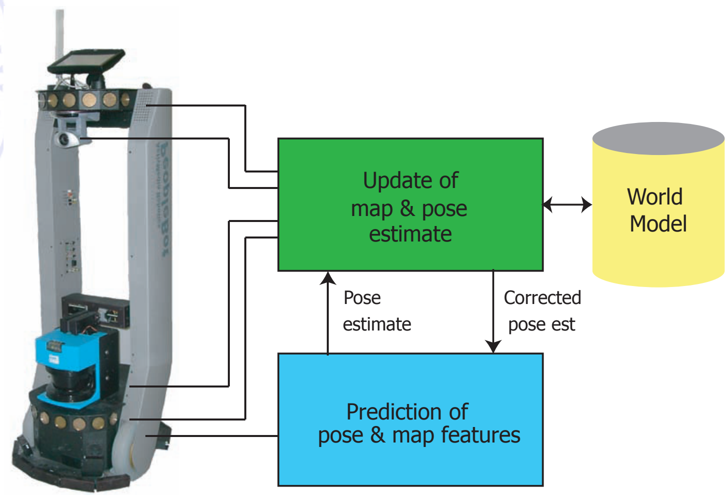
Outline

Outline of problem



Ensure the robot does not get lost!

The basics for SLAM

- State of robot is modelled as the pose

$$\vec{x}_R = \left[\begin{array}{ccc} x & y & \theta \end{array} \right]^T$$

- Map features can be represented as points or lines, i.e.:

$$\vec{x}_i = \left[\begin{array}{cc} x_i & y_i \end{array} \right]^T$$

Estimation as a Kalman Problem

- Prediction by odometric modelling
- Updating as a Kalman process, with the state

$$\vec{x}_{state} = \begin{bmatrix} \vec{x}_R \\ \hline \vec{x}_1 \\ \vdots \\ \vec{x}_n \end{bmatrix}$$

Why is SLAM difficult?

- The number of map hypotheses is very large
- Often the signal to noise ratio for features is ≈ 1
- Robust discriminative features are not common
- The "process" is often approximated

Problems?

1. Flexible inclusion/exclusion of measurements?
2. Handling of linearization?
3. Dealing with topological constraints?
   - Loop closing etc.

Data handling

- Easy inclusion and/or exclusion of data at any time in the process.
- How to avoid too early a commitment to a particular map hypothesis.
- Design of a representation that allow any-time inclusion/exclusion of data?

Linearizations?

- Linearization might cause divergence in the data.
- Reported by several, e.g. **??**
- Consistent handling of non-linearities
  - Start by exact handling of non-linearities
  - As data matures a linearization is permitted
  - Identification of major non-linearities to include them

Topological constraints?

- Consistent inclusion of topological constraints
- Two step strategy:
  1. Close approximation of system in a trivial way
  2. Fine tune full model by adding “smaller” corrections

Outline

Problem statement

- $\{x_i\}$ the robot path (set of poses), $(i \in \{1 \ldots N_p\})$
- $\{z_j\}$ feature coordinates $(j \in \{1..N_m\})$
- $\{d_i\}$ dead reckoning measurements, between feature measurements
- $\{f_k\}$ feature measurements, $(k \in \{1..N_f\})$
- Λ the $f \leftrightarrow z$ association

$$P(x, z, d, f, \Lambda) = P(d, f | x, z, \Lambda) P(x, z, \Lambda)$$

Probabilistic model

$$
\begin{aligned}
P(x,z,d,f,\Lambda) &\propto P(d,f|x,z,\Lambda)P(x,z,\Lambda) \\
P(d,f|x,z,\Lambda) &\propto P(d|x)P(f|x,z,\Lambda) \\
P(x,z,\Lambda) &\propto P(\lambda) = P(N_f) \propto e^{-\lambda N_f} \\
P(x,z,d,f,\Lambda) &\propto P(d|x)P(f|x,z,\Lambda)e^{-\lambda N_f}
\end{aligned}
$$

An energy model

- Definition of energy/entropy of the model:

$$E(x,z,d,f,\Lambda) = -\log(P(d|x)) - \log(P(f|x,z,\Lambda)) + \lambda N_f$$

- Or $E(x,z,d,f,\Lambda) = E_d + E_f + E_\Lambda$
- Or: ...

An energy model

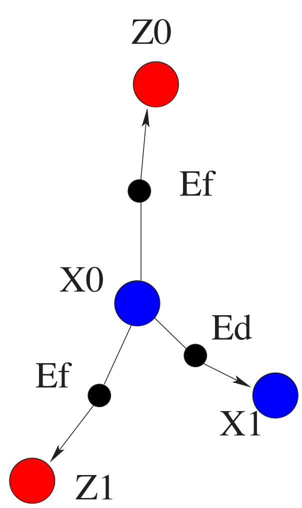
$$
\begin{aligned}
E(x, z, d, f, \Lambda) &= E_d(x) + E_f(x, z) + E_\Lambda(n_j) && (1)\\
E_d &= -\sum_{i=1}^{N_p} log(P(d_i|x_{i-1}, x_i)) = \frac{1}{2}\sum_{i=1}^{N_p} \xi_i^T k_i \xi_i && (2)\\
E_f &= -\log(P(f|x, z, \Lambda)) = \frac{1}{2}\sum_{k=1}^{N_m} \eta_k^T k_k \eta_k && (3)\\
E_\Lambda &= -\sum_{j=1}^{N_f} \lambda(n_j - 1) && (4)\\
\xi_i = T(x_i|x_{i-1}) - d_i & \qquad \eta_k = h(T(z_j|x_i)) - f_k
\end{aligned}
$$

Outline

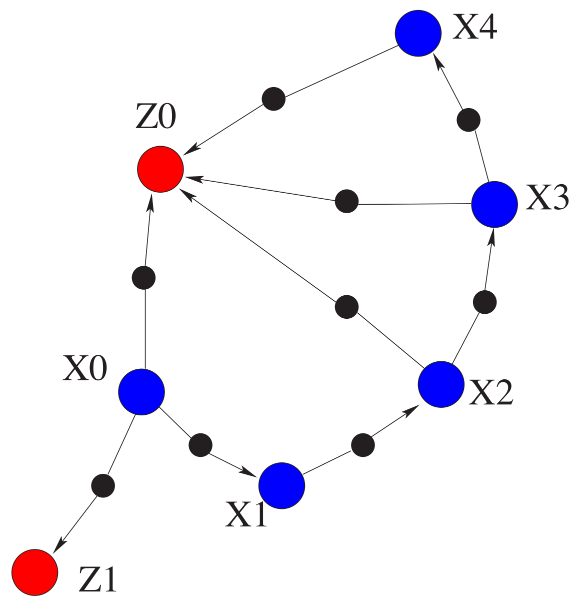
Organizing a model

- Graph representation
- Two types of nodes:
  1. State notes
     - Poses (x_i)
     - Features (z_j)
  2. Energy Nodes (Computation of Eqn (1))
     - Connected to the state nodes needed for computation
     - Movement (d_i, k_i)

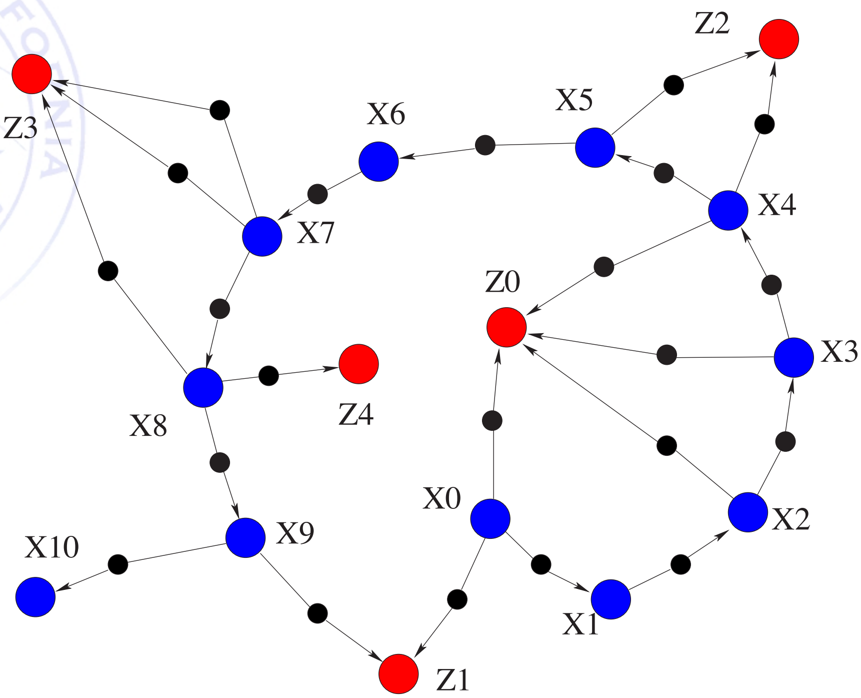
Starting a model



Entering more data



A graphical model example



Map updating

- Optimal solution to Eq. (1): $(\mathrm{argmin} E)$ is not realistic.
- Relaxation techniques allow iterative updating
- In a time step:
  1. Add a new state node (pose)
  2. Any new features/measurements?
  3. Update the rest of the map – minimize energy

Map Update–Energy

- Eqn (1) Taylor expanded for a node A:

$$\begin{aligned} E_A &= \sum_{i \in edge(A)} [E_i(\bar{x}_A, \bar{x}_i) + \nabla E_i(\bar{x}_A, \bar{x}_i) \begin{pmatrix} \Delta x_A \\ \Delta x_i \end{pmatrix} \\ &+ \frac{1}{2} \begin{pmatrix} \Delta x_A & \Delta x_i \end{pmatrix} \nabla^T \nabla E_i(\bar{x}_A, \bar{x}_i) \begin{pmatrix} \Delta x_A \\ \Delta x_i \end{pmatrix}] \end{aligned}$$

Notation:

$$\begin{aligned} \mathcal{G} &= \nabla E_A = \begin{pmatrix} \mathcal{G}_A \\ \mathcal{G}_i \end{pmatrix} \\ \mathcal{H} &= \nabla^T \nabla E_A = \begin{pmatrix} \mathcal{H}_{AA} & \mathcal{H}_{Ai} \\ \mathcal{H}_{Ai} & \mathcal{H}_i i \end{pmatrix} \end{aligned}$$

Map Update–Energy (II)

- Optimizing a node A:

$$(x_A - \bar{x}_A) = -\mathcal{H}_{AA}^{-1}\mathcal{G}_A \tag{5}$$

- With an energy change of

$$\Delta E_A = \frac{1}{2}\mathcal{G}_A^T\mathcal{H}_{AA}^{-1}\mathcal{G}_A \tag{6}$$

Map Update–Energy (III)

- ΔE_A decides on update strategy:
    - Steepest decent – close to saddle point
    - Use eqn. (5) direct
    - Chained update
        - Locate a node that is "good"
        - Update from that node

Feature Matching

- λ decides on the value of new measurements
  1. Add a new feature
  2. Compute energy change
  3. If change too large, remove association
- Similar to EM (**?**)
- Matching/graph updating anytime

Example

- ATRV Vehicle with Trimble DGPS 212
- Sick LMS 291 Laser scanner
- CrossBow DFOG INS package
- State $(\theta_n, x_n, y_n)^T$
- Lines w. end-points (x, y)
- See **?** for details
- Operating around a house.
- 7500 Pose Nodes, 11975 line measurements
- Update time 30 ms (550 MHz Pentium)

SLAM example

Outline

Status?

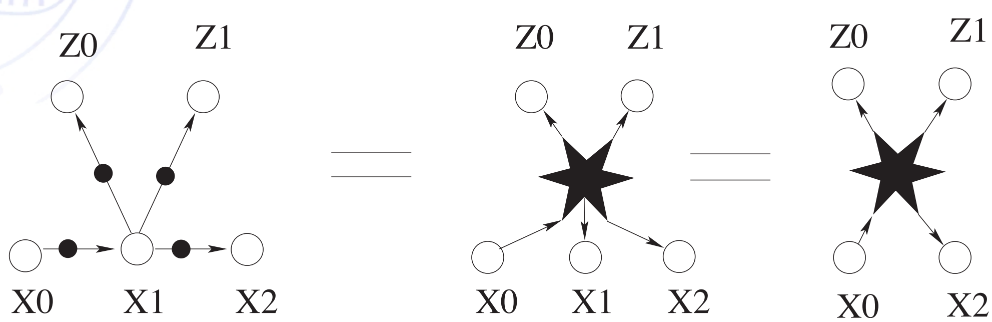
- Flexible association of data, anytime
- A framework to avoid linearization effects
- Loop closing:
  1. Recognition of loop closure (place recognition)
  2. Updating map to include the topological constraint
- Doing 2) often has limited effect
- A proposal . . .

Outline

Reducing the graph

- How can state nodes be removed from the graph?
- In a linear system one could generate a closed form solution
- In a non-linear case an approximation can be used.
- Consider elimination of a node A given neighboring nodes B.
- Consider the energy from state A expanded.

The basic idea



The Energy from A

$$
\begin{aligned}
E_A &= \sum_{j \in edge(A)} [E_i(\bar{x}_A, \bar{x}_j) + \mathcal{G}_j(x_j - \bar{x}_j) \\
&+ \frac{1}{2} \begin{pmatrix} x_A - \bar{x}_A & x_j - \bar{x}_j \end{pmatrix} \mathcal{H}_j(\bar{x}_A, \bar{x}_j) \begin{pmatrix} x_A - \bar{x}_A \\ x_j - \bar{x}_j \end{pmatrix}]
\end{aligned} \tag{7}
$$

Transforming to B:

$$
(x_A - \bar{x}_A) = -\mathcal{H}_{AA}^{-1} \mathcal{H}_{AB} (x_B - \bar{x}_B)
$$

Elimination of A

- Substitution into eqn (7) eliminates A

$$
\begin{aligned}
E^* &= \sum_{j \in edge(A)} [E_j(\bar{x}_A, \bar{x}_j) + \mathcal{G}_j(x_j - \bar{x}_j) \\
&+ \frac{1}{2} (x_j - \bar{x}_j)^T \mathcal{H}_{jj} (x_j - \bar{x}_j) \\
&- \sum_{i \in edge(A)} (x_i - \bar{x}_i)^T \mathcal{H}_{iA} \mathcal{H}_{AA}^{-1} \mathcal{H}_{Aj} (x_j - \bar{x}_j)]
\end{aligned}
$$

- New term connects i and j (new connection)

Star nodes

- The fused nodes are termed *Star* nodes.
- In principle the entire graph could be fused into a single node, as done in EKF.
- In practice star nodes are used to fuse local "maps"
- Typically upto 128 state nodes are considered for "fusion".
- The star nodes are considered retroactively after say 50 poses.

Making star nodes invariant

- Star nodes updates energy using the Hessian $\mathcal{H}$
- If measurements have symmetries (such as lines) $\mathcal{H}$ will have zero eigenvalues.
- Project state nodes to "natural coordinates" (q) in a lower dimensional space without symmetries.

$$q_i = PT(x_i|x_0)$$

Mapping to natural coordinates

- In general:

$$\mathcal{H}_{xx} = J^T \mathcal{H}_{qq} J + \frac{\partial^2 q}{\partial x \partial x} \mathcal{G}_q \approx J^T \mathcal{H}_{qq} J$$

- Thus:

$$\mathcal{H}_{qq} = \tilde{J} \mathcal{H}_{xx} \tilde{J}^T$$

- Using SVD it is possible to find eigenvalues b_j and eigenvectors V_j of the Hessian

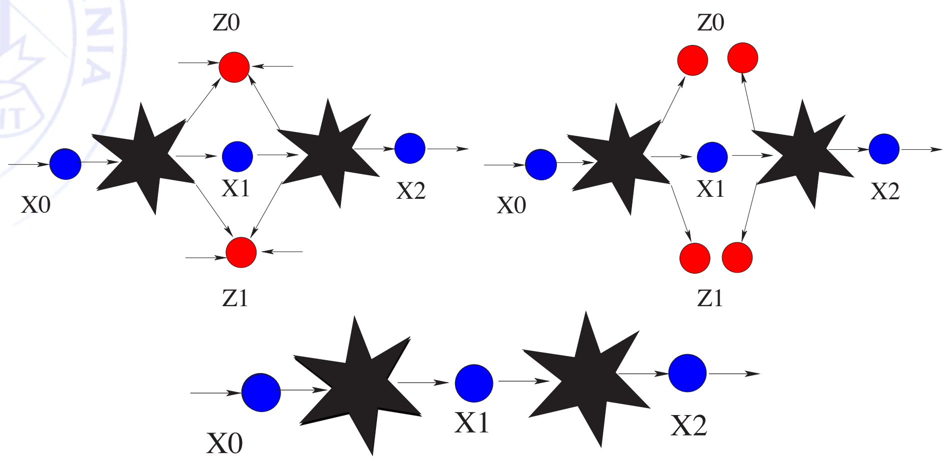
Energy & Properties

- Using the eigenvectors the energy at $\bar{q}$ is now:

$$E^* = E^*(\bar{q}) + \frac{1}{2} \sum_j b_j (V_j \Delta q)^2$$

- Recentered nodes are linear in energy
- A state node connected to only one star node can be eliminated ($\Delta q = 0$)
- Star nodes like local maps in Atlas (**?**)

Optimizing global calculations



Outline

Closing loops

- With $\Delta q = 0$ the state nodes between star nodes is ignored.
- Define a cost function using Lagrange multipliers

$$C(\Lambda, \Delta q) = \Lambda(\sum_i \Delta x - d_c) + \frac{1}{2}\sum_i \sum_j b_{ji}(V_{ji}\Delta q)^2$$

- Where i is the star node index, Δx_i is the pose difference between the poses. d_c is the pose constraint.
- I.e.

$$\Delta x_i = \begin{pmatrix} \mathcal{R}_i & 0 \\ 0 & 1 \end{pmatrix} (\Delta q_i + \bar{q}_i)$$

Solving for closed loops

- Linearisation gives:

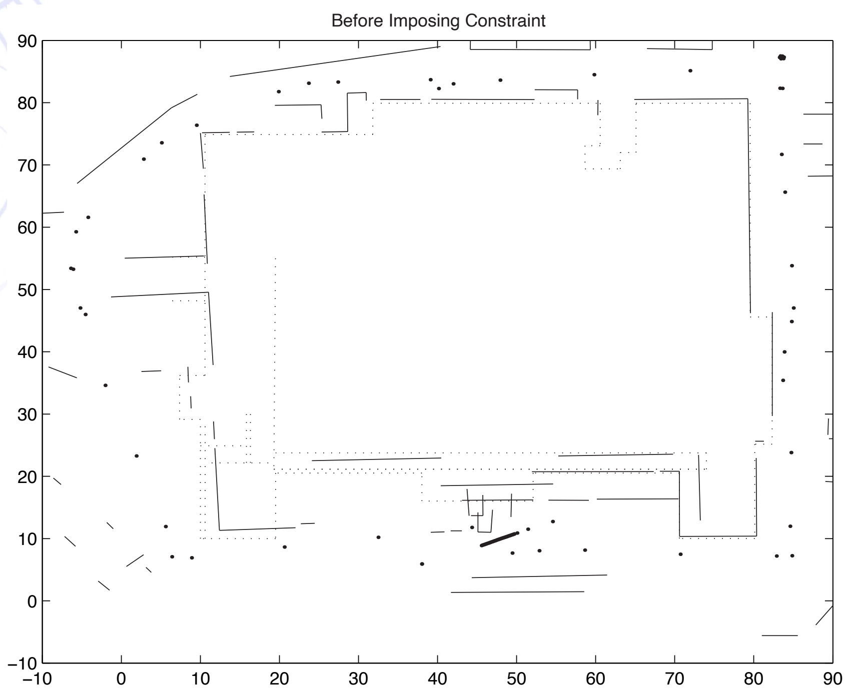
$$\Delta q_i = -\sum_j \frac{V_{ji}^T V_{ji}}{b_j} \begin{pmatrix} \mathcal{R}_i & 0 \\ 0 & 1 \end{pmatrix} \Lambda^T$$

- Where:

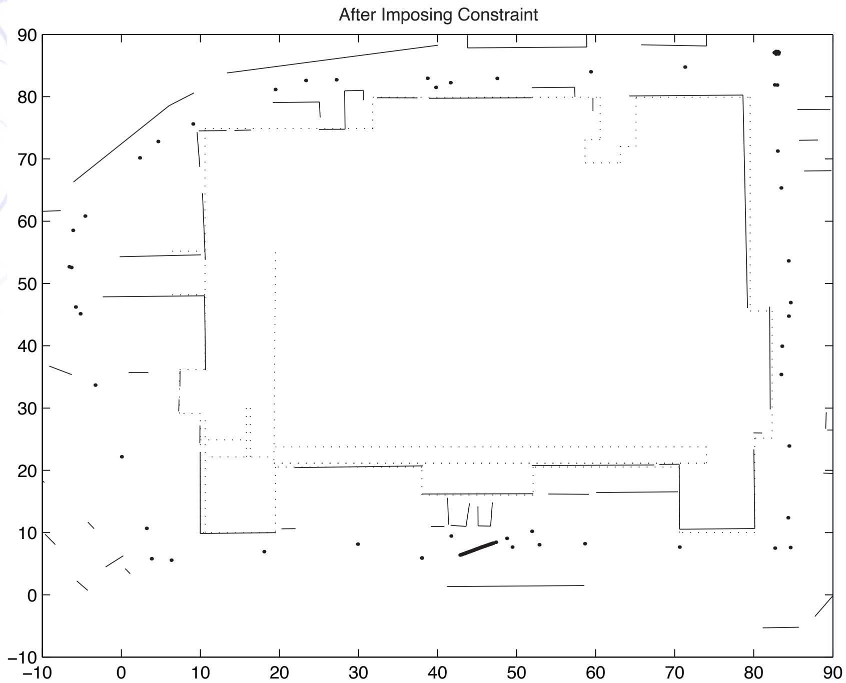
$$\begin{aligned} \Lambda^T &= S^{-1}\left\{\left[\sum_i \begin{pmatrix} \mathcal{R}_i & 0 \\ 0 & 1 \end{pmatrix} \bar{q}_i\right] - d_c\right\} \\ S &= \sum_i \left\{\begin{pmatrix} \mathcal{R}_i & 0 \\ 0 & 1 \end{pmatrix} \left[\sum_j \frac{V_{ji}^T V_{ji}}{b_j}\right] \begin{pmatrix} \mathcal{R}_i & 0 \\ 0 & 1 \end{pmatrix}\right\} \end{aligned}$$

Outline

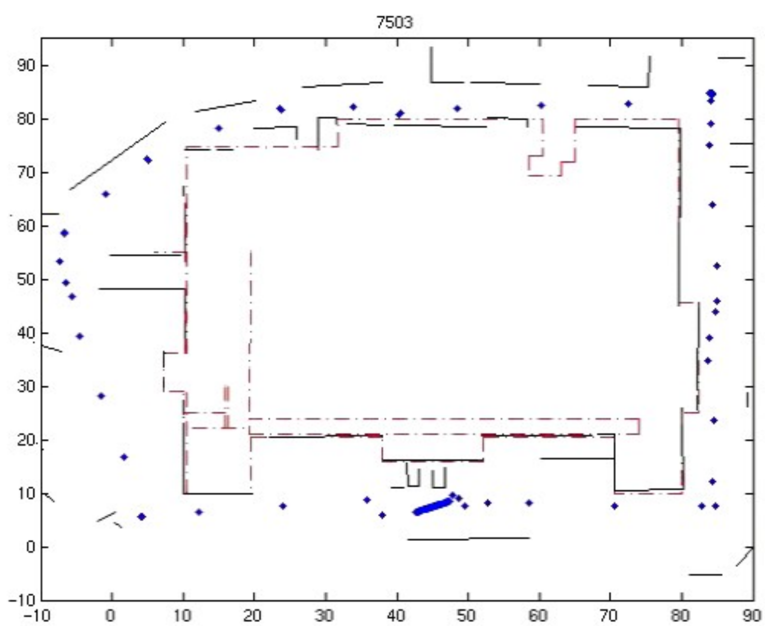
Map example - no closing



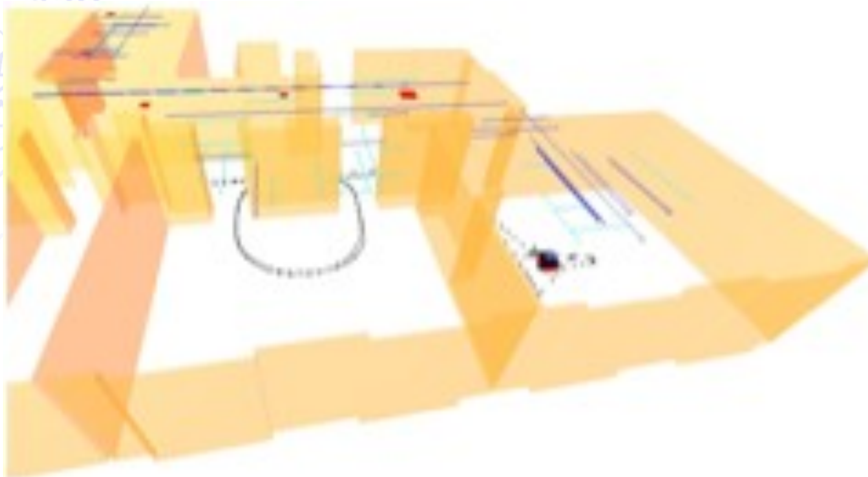
Map example - closing



Example Movie



Example Movie 2



Closing loops

- Loop closing has a complexity of $O(N+1)$ where N is the number of star nodes.
  - Once loop closing is achieved
  - Turn back on inter-star relation to fine tune
- Achieved in 1-2 seconds for large environments
- Constraints similar to "strong links" in **?**.

Outline

Summary

- A new efficient representation for SLAM
  - Easy integration of data anytime
  - "Chunk-ing" data into local maps (star nodes)
  - Addition of constraint for loop closing
- Computationally efficient (10-12 ms / pose)
- Losing closing without linearization problems
- Further details in **????**