## CSE276A - Visual Servoing

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## HW2 - Closing the loop with vision

- Due: 31 October @ midnight
- Two options:
- Use AprilTags as landmarks in the environment
- Use natural landmarks for extra credit (it is hard!)
- Tutorial on how to use DL for detection of objects say Yolo
- https://pyimagesearch.com/2018/11/12/yolo-object-detection-withopencv/
- 20\% extra credit
- Use detected landmarks to estimate your own position
- Correct for drift in the control of the vehicle


## AprilTags


https://github.com/AprilRobotics/apriltag

## Practical stuff

- You have to calibrate your camera!
- OpenCV see later
- ROS camera calibration
- Example process with ROS modules


| Visual Servoing |
| :--- |
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## Literature

- Peter Corke, Robotics - Vision and Control, Springer Verlag, 2nd Edition, 2017 - Chapter 15

Types of visual serving

- Control Strategy
- Position based servoing
- Image based servoing

- Camera Placement
- Eye-in-Hand
- Stand-along



## Process



## Categorization



Utilization of kinematic models



Position based visual serving


Control

- Error Metric

$$
\begin{aligned}
\Delta^{R} \mathbf{t}_{G} & ={ }^{R} \mathbf{t}_{G}-{ }^{R} \mathbf{t}_{G}^{*} \\
\Delta^{R} \theta_{G} & ={ }^{R} \theta_{G}-{ }^{R} \theta_{G}^{*}
\end{aligned}
$$

- Relevant Coordinate frames

$$
{ }^{R} \mathbf{X}_{G}^{*}={ }^{R} \mathbf{X}_{C}{ }^{C} \mathbf{X}_{O}{ }^{O} \mathbf{X}_{G}^{*}
$$

## PBS Cartoon



## Strategy

- Compute

$$
\mathbf{e}=\left[\begin{array}{c}
\Delta^{R} \mathbf{t}_{G} \\
\Delta^{R} \theta_{G}
\end{array}\right]
$$

- The change in robot coordinates (use, PID, ...)

$$
\dot{\mathbf{q}} \approx \mathbf{K e}
$$

Expanding the error metrics

- Expanding the transformation

$$
\begin{aligned}
{ }^{R} \mathbf{t}_{G}^{*} & ={ }^{R} \mathbf{R}_{C}{ }^{C} \hat{\mathbf{R}}_{O}{ }^{o} \mathbf{t}_{G}^{*}+{ }^{R} \mathbf{R}_{C}{ }^{C} \hat{\mathbf{t}}_{O}+{ }^{R} \mathbf{t}_{C} \\
{ }^{R} \Omega_{G}^{*} & ={ }^{R} \Omega_{C}+{ }^{R} \mathbf{R}_{C}{ }^{C} \hat{\Omega}_{O}+{ }^{R} \mathbf{R}_{C}{ }^{C} \hat{\mathbf{R}}_{O}{ }^{o} \Omega_{G}^{*}
\end{aligned}
$$

- Integrating
${ }^{R} \theta_{G}^{*} \approx{ }^{R} \theta_{C}+{ }^{R} \mathbf{R}_{C}{ }^{C} \hat{\theta}_{O}+{ }^{R} \mathbf{R}_{C}{ }^{C} \hat{\mathbf{R}}_{O}{ }^{O} \theta_{G}^{*}$
- Substituting from above

$$
\begin{aligned}
\Delta^{R} \mathbf{t}_{G} & ={ }^{R} \mathbf{t}_{G}-{ }^{R} \mathbf{t}_{C}-{ }^{R} \mathbf{R}_{C} C \hat{\mathbf{t}}_{O}-{ }^{R} \mathbf{R}_{C}{ }^{C} \hat{\mathbf{R}}_{O} O_{\mathbf{t}_{G}^{*}} \\
\Delta^{R} \theta_{G} & \approx{ }^{R} \theta_{G}-{ }^{R} \theta_{C}-{ }^{R} \mathbf{R}_{C} C^{C} \hat{\theta}_{O}-{ }^{R} \mathbf{R}_{C}{ }^{C} \hat{\mathbf{R}}_{O}{ }^{o} \theta_{G}^{*}
\end{aligned}
$$

Simulated camera


Example PBS simulation


Canonical image motion patterns



Image Based Visual Servoing (IBVS)


The Image Jacobian

- How does features move when we move the robot?

$$
\dot{\mathbf{f}}=\mathbf{J} \dot{\mathbf{q}}
$$

- Image Jacobian

$$
J(\mathbf{q})=\left[\frac{\delta \mathbf{f}}{\delta \mathbf{q}}\right]=\left[\begin{array}{ccc}
\frac{\delta f_{1}(\mathbf{q})}{\delta q_{1}} & \ldots & \frac{\delta f_{1}(\mathbf{q})}{\delta q_{m}} \\
\vdots & \ddots & \vdots \\
\frac{\delta f_{k}(\mathbf{q})}{\delta q_{1}} & \ldots & \frac{\delta f_{k}(\mathbf{q})}{\delta q_{m}}
\end{array}\right]
$$

Going back to basics

$$
\begin{gathered}
x=\frac{X}{Z}, y=\frac{Y}{Z} \\
\dot{x}=\frac{\dot{X} Z-X \dot{Z}}{Z^{2}}, \dot{y}=\frac{\dot{Y} Z-Y \dot{Z}}{Z^{2}} \\
\dot{\boldsymbol{P}}=-\boldsymbol{\omega} \times \boldsymbol{P}-\boldsymbol{v} \\
\dot{X}=Y \omega_{z}-Z \omega_{y}-v_{x} \\
\dot{Y}=Z \omega_{x}-X \omega_{z}-v_{y} \\
\dot{Z}=X \omega_{y}-Y \omega_{x}-v_{z}
\end{gathered}
$$

For K points we can stack

$$
\left[\begin{array}{c}
\dot{u}_{1} \\
\dot{v}_{1} \\
\vdots \\
\dot{u}_{k} \\
\dot{v}_{k}
\end{array}\right]=\mathbf{A}\left[\begin{array}{cccccc}
\frac{1}{Z_{1}} & 0 & -\frac{x_{1}}{Z_{1}} & -x_{1} y_{1} & 1+x_{1}^{2} & -y_{1} \\
0 & \frac{1}{Z_{1}} & -\frac{y_{1}}{Z_{1}} & -1-y_{1}^{2} & x_{1} y_{1} & x_{1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{1}{z_{k}} & 0 & -\frac{x_{k}}{Z_{k}} & -x_{k} y_{k} & 1+x_{k}^{2} & -y_{k} \\
0 & \frac{1}{Z_{k}} & -\frac{y_{k}}{Z_{k}} & -1-y_{k}^{2} & x_{k} y_{k} & x_{k}
\end{array}\right]\left[\begin{array}{c}
V_{X} \\
V_{Y} \\
V_{Z} \\
\omega_{X} \\
\omega_{Y} \\
\omega_{Z}
\end{array}\right]
$$

Going back to basics

- With a unit focal length we would have

$$
\begin{aligned}
{ }^{C} \dot{\mathbf{P}} & ={ }^{C} \Omega \times{ }^{C} \mathbf{P}+{ }^{C} \mathbf{V} \\
{\left[\begin{array}{c}
\dot{u} \\
\dot{v}
\end{array}\right] } & =\left[\begin{array}{cc}
k_{u} & 0 \\
0 & k_{v}
\end{array}\right]\left[\begin{array}{cccccc}
\frac{1}{z} & 0 & -\frac{x}{z} & -x y & 1+x^{2} & -y \\
0 & \frac{1}{z} & -\frac{y}{z} & -1-y^{2} & x y & x
\end{array}\right]\left[\begin{array}{c}
V_{X} \\
V_{Y} \\
V_{Z} \\
\omega_{X} \\
\omega_{Y} \\
\omega_{Z}
\end{array}\right]
\end{aligned}
$$

- We can define an error metric

$$
\begin{aligned}
\mathbf{e}(\mathbf{f}) & =\mathbf{f}^{c}-\mathbf{f}^{*} \\
\mathbf{u} & =\dot{\mathbf{q}}=\mathbf{K} \mathbf{J}^{\dagger} \mathbf{e}(\mathbf{f})
\end{aligned}
$$

Visual impression of image jacobian


Simulated example


Sketch of an IBVS example

2.5D image based servoing


What if we could estimate $Z$ online?

$$
\begin{aligned}
\binom{\dot{u}}{\dot{v}} & =\left(\begin{array}{ccc|ccc}
-\frac{f}{\rho_{u} Z} & 0 & \frac{\bar{u}}{Z} & \frac{\rho_{u} \overline{u v}}{f} & -\frac{f^{2}+\rho_{u}^{2} \bar{u}^{2}}{\rho_{u} f} & \bar{v} \\
0 & -\frac{f}{\rho_{v} Z} & \frac{\bar{v}}{Z} & \frac{f^{2}+\rho_{v}^{2} v^{2}}{\rho_{v}} & -\frac{\rho_{v} \overline{u v}}{f} & -\bar{u}
\end{array}\right)\binom{\boldsymbol{v}}{\omega} \\
& =\left(\begin{array}{l}
\left.\left.\frac{1}{Z} \boldsymbol{J}_{t} \right\rvert\, \boldsymbol{J}_{\omega}\right)\binom{\boldsymbol{v}}{\omega} \\
\end{array}+\frac{1}{Z} \boldsymbol{J}_{t} \boldsymbol{v}+\boldsymbol{J}_{\omega} \omega\right.
\end{aligned}
$$

- We can rearrange

$$
\left(J_{t} v\right) \frac{1}{Z}=\binom{\dot{u}}{\dot{v}}-\boldsymbol{J}_{\omega} \omega \quad \Rightarrow \quad \mathbf{A} \boldsymbol{\theta}=\mathbf{b}
$$

## Summary / Take Home

- Image vs. position based servoing
- Derivation of motion impact on image coordinates
- Use of basic motion models to control the robot
- Great support for PBS and IBVS in the RVT toolkit

Example from the book


